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PROBABILITY
AND
INDUCTION

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BY

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PREFACE

THIS book is based on lectures I have given to students of philosophy, and it is intended for readers who are interested in the philosophical problems suggested by the title. I do not pretend that it is either a treatise on the mathematical theory of probability or a practical guide to scientific method. For I have not the ability to produce the former, and I do not think it is the business of a philosopher (or perhaps of anyone) to try to provide the latter. So far as mathematics and natural science are concerned, I shall be content if I have avoided howlers.

The philosophical problems discussed are elementary in the sense that they have to do with first principles, and I have tried to make my treatment of them elementary also in the other sense of the word, that is to say, intelligible without much previous reading about the subject. But some of the statements to which I have committed myself are very controversial, and it may be useful to make clear that I do not attach equal importance to them all. In Part I, for example, I have written of knowledge as though it were indefinable, but this is merely because I have seen no satisfactory analysis of knowledge and do not think it necessary for my present purpose to try to find one. In spite of what I have said, I should welcome a new attempt to analyse this notion. At the other extreme, the general account of induction given in Part IV seems to me substantially correct, and I wish to stand by it. The theory of natural necessity in Part II and the range theory of probability in Part III come between these contentions in order of importance. I am acutely conscious of the difficulties of my views and the insufficiency of my arguments, and yet I cannot at present see any other way of describing matters which seems at all plausible. If I am mistaken in what I have said about these topics, I hope that I have at least written clearly enough to be found out quickly.

In accordance with the conventions of the Clarendon Press, logical and mathematical symbols have been printed without quotation marks even where they are themselves the subjects of discourse. I hope no reader will be seriously distressed by this usage, which is almost universal in mathematical texts. In certain contexts it can lead to dangerous confusion, and my own preference is for a rigorous distinction between use and mention at all times ;

but no real ambiguity results here from the omission of inverted commas, and I have therefore not felt justified in pressing for their restoration at the expense of much valuable labour.

My debt to Lord Keynes's *Treatise on Probability* is obvious even where I criticize his doctrines. On the history of induction, or rather of philosophical views about induction, I have learnt much from *Les Théories de l'induction et de l'expérimentation* by M. André Lalande. To my friends Professor Gilbert Ryle, Dr. Friederich Waismann, and Dr. Karl Popper I owe a great deal that I cannot now disentangle from my own thought; but I am afraid they may be surprised and shocked by my conclusions. The fact that I have not made full use of some important recent contributions to the subject, in particular those of Professor R. Carnap and Professor G. H. von Wright, is due to the circumstances in which I have written this book. The first draft was begun in the summer of 1939, laid aside during the war while I was engaged on other work, and finished in the short period of comparative ease before the universities became crowded with men returning from military service. Since the autumn of 1946 I have had no time to do more than remove some of the faults which became evident when the whole was in typescript.

I wish to thank Professor Henry Price very warmly for the great care with which he read my work and advised me about the revision of it. His kindness has saved me (and the reader) from some silly passages and many that were obscure.

Finally, I wish to dedicate this book to my wife, who has helped me with advice and encouragement throughout the making of it.

July 1948

W. K.

SOME misprints and mistakes (including mathematical errors in §§ 28, 43, and 45) have been corrected for the second impression, but I have not tried to remove any defects of philosophical argument, since that could not be done without extensive rewriting.

June 1951

W. K.

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PART I

INTRODUCTORY: KNOWLEDGE AND BELIEF

§ I. THE SCOPE OF THE INQUIRY

THE extent of our knowledge is less than we could wish. It may perhaps be wider than some philosophers have supposed, but it is clearly not wide enough to enable us to answer with certainty all the questions that arise in the practical affairs of everyday life and the still more numerous questions that puzzle us when we study history or science. Probability may be described as the substitute with which we try to make good the shortcomings of our knowledge. It does not fill the gap entirely, for there are many questions about which we cannot even form opinions; but it often enables us to act rationally when without it we should be reduced to helplessness, and it gives at least some satisfaction to our intellectual curiosity. This was perhaps the meaning of Bishop Butler's famous remark, 'To us, probability is the very guide of life.'¹ He was contrasting our state with that of an infinite intelligence which could discern 'each possible object of knowledge, whether past present or future, . . . absolutely as it is in itself'; and he wished to argue that, as in the common pursuits of life we rely on a kind of reasoning which can provide only probable conclusions, so too in theology we should be prepared to make tentative arguments from experience. I do not know whether his method is in favour now among theologians, but there can be no doubt of the importance which empirical scientists attach to it.

The variety of the situations in which we use the notion of probability is illustrated by the following sentences, each of which contains the word 'probable' or one of its derivatives:

- (a) It is probable that there will be rain before the day is over.
- (b) It is very improbable that a man with testimonials as good as these will fall into dishonesty.
- (c) Stonehenge was probably built for use as a temple.
- (d) If Hannibal had marched on Rome, he would probably have taken it.
- (e) We know now that the stories which Marco Polo told on his return to Venice were true, however improbable they may have been for his contemporaries.

¹ *Analogy of Religion*, Introduction, § 4.

- (f) The probability of throwing a number greater than four with a true die is 1:3.
- (g) Statistics indicate that if a wounded man is treated immediately with penicillin the probability of his escaping sepsis is more than 9:10.
- (h) We cannot assign high probabilities to the generalizations made in sociology, because the number of cases in which we are able to confirm them is not very large.
- (i) The probability of the atomic theory of matter has been greatly increased by the evidence which physicists and chemists have collected during the past century.

In ordinary life we often have occasion to make remarks like (a) and (b), and we often meet statements such as (c), (d), and (e) when we read works on history. Example (e) is worth special notice, because it shows that what is improbable may nevertheless be true. Although as plain men we do not often try to make precise numerical estimates of probability, we all recognize that statements like (f) and (g) are needed in various branches of science and in certain specialisms such as actuarial work. Examples (h) and (i) are interesting as specimens of the way in which we pass judgement on scientific generalizations and explanatory hypotheses.

Our dependence on the notion of probability is not confined, however, to those cases in which we employ the word 'probable' or one of its derivatives to state our views, for there are other ways of expressing the same thought. Sometimes we speak of the balance of chances. At other times such words as 'likely', 'reliable', 'trustworthy' seem more appropriate. And we must admit on reflection that in many cases in which we do not ordinarily use the word 'probable' or any equivalent expression it would be wiser to do so if there were any danger of misunderstanding. I may, for example, assert without qualification that Julius Caesar landed on the south coast of England and even count this as an item of my knowledge, but if I am pressed to say whether I know it for certain, I can only reply that I have it on good authority and consider it extremely probable or almost certain. It is clear, therefore, that the realm of probability is very large indeed and covers even much of what we loosely call our knowledge.

In this book I wish to consider the philosophical theory of probability. The subject has also been investigated by mathematicians, and I shall have to say something about their work, but

only in so far as it is relevant to the discussion of the philosophical problems. How the philosophical problems arise and how we should try to solve them will become clear, I hope, in the course of the book. I do not propose, therefore, to say anything here in general terms about the aim and method of philosophical inquiries; but it will be obvious already that the philosophical theory of probability is part of epistemology, that is to say, of the philosophical discipline in which we study the different kinds of knowledge and related topics such as the nature of belief.

§ 2. SUBJECTIVIST THEORIES OF PROBABILITY: MEANINGS OF 'BELIEF'

If, as seems natural, we start by contrasting probability statements with statements in which we express knowledge, the question immediately arises: 'What then do we express by probability statements?'

One of the commonest ways of introducing the notion of probability into discourse is by means of an adverb. We may say, for example, 'It is probably raining in the Hebrides.' In late antiquity any statement which included an adverb of this kind or any similar expression (e.g. 'it is probable that . . .' or 'it is possible that . . .') was classified as *modal*, and it is now the custom for logicians to use the name 'modality' for that division of their study in which they treat of necessity, possibility, and probability. This terminology is not illuminating, and may even be misleading. In what sense does the adverb 'probably' signify a mode or manner? Clearly it is not used in the same way as ordinary adverbs, which qualify verbs much as adjectives qualify nouns. When I say that it is raining heavily, I mean that it is raining in a special way, but when I say that it is probably raining, I certainly do not want to suggest that there is a special mode or manner of raining which I call 'probably raining'. Such an interpretation is so obviously absurd that it has never been seriously defended. On the contrary most persons who give any thought to the matter are inclined to jump immediately to the opposite extreme and say that probability must be subjective.

One subjectivist theory which has found its way into many of the older text-books of logic is presented as a doctrine about different modes or manners of assertion. It is argued that a man who utters the sentence 'It is probably raining' is asserting the proposition that it is raining but doing so in a special fashion or

4 SUBJECTIVIST THEORIES OF PROBABILITY: 'BELIEF'

with a special qualification, much as a man who says 'Unfortunately it is raining' may be held to assert the proposition that it is raining but with an additional comment about his own state of mind in making the assertion. It is difficult to attach any precise meaning to the phrase 'mode or manner of assertion' as it is used in this theory; but we need not trouble ourselves about the matter, for the doctrine seems to be founded on the mistaken assumption that any sentence which contains one of the modal adverbs must be taken as an assertion of the proposition which would ordinarily be asserted by the use of the sentence without the adverb. This assumption is only plausible when we are dealing with modal statements which contain the adverb 'necessarily'.¹ If I say 'It is necessarily raining', I am indeed committed to the assertion that it is raining. But if I say 'It is probably raining', I am not asserting in any way that it is raining, and the discovery that no rain was falling would not refute my statement, although it might render it useless. The mistake seems to be due to overmuch concentration on the adverbial expression of modality. In order to escape from it we need only remember that 'It is probably raining' is equivalent to 'It is probable that it is raining'. In the second formulation we have no excuse for assuming an assertion that it is raining, since the words 'it is raining' occur here only as they do in 'It is false that it is raining', i.e. as a subordinate clause. In short, the view that probability is a mode of assertion is derived from the same source as the view that probability is a mode of being, namely, from a failure to see that modal adverbs function in a quite peculiar way.

The most common subjectivist theory of probability is based on the very different assumption that a probability statement is really an assertion about the speaker's own state of mind. According to this doctrine probability is neither a mode of being nor a mode of assertion. It belongs to propositions (i.e. thinkables or assertable contents), but not as an intrinsic property, for it is simply the degree of belief which we attach to them. James Bernoulli, who made great contributions to the mathematical theory of probability, appears to have held some such view, and he has been followed by a number of other distinguished persons. We must therefore examine the suggestion carefully.

Having started with an antithesis between probability and knowledge, it is natural that we should go on to connect proba-

¹ Or 'actually', if that is allowed to be modal.