

Wei Liu and Stephan Weiss



# Wideband Beamforming

Concepts and Techniques



Wiley Series on  
Wireless Communications  
and Mobile Computing



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# **WIDEBAND BEAMFORMING**

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# WIDEBAND BEAMFORMING CONCEPTS AND TECHNIQUES

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*In memory of my mother*

Wei Liu



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# Preface

Beamforming is a spatial filtering technique for receiving signals illuminating an array of sensors from some specific directions, whilst attenuating signals from other directions. Depending on the signal bandwidth, it can be divided into two categories: narrowband beamforming and wideband beamforming. For narrowband beamforming, it is achieved by an instantaneous linear combination of the received array signals. However, when the involved signals are wideband, we have to employ an additional processing dimension for effective operation, such as tapped delay-lines (or FIR/IIR filters), or the recently proposed sensor delay-lines, which lead to a wideband beamforming system.

Wideband beamforming has been studied extensively in the past due to its applications in various areas ranging from radar, sonar, microphone arrays, radio astronomy, seismology, medical diagnosis and treatment, to communications. In particular, since speech/sound is a natural source of wideband signals, much of the research and development in wideband beamforming has been focused on the area of microphone arrays.

Traditionally, beamforming is considered as part of the wider area of array signal processing and chapters relating to beamforming can be found in many books on array signal processing. Recently, due to its importance in the wireless communications area, there have been some books dedicated to beamforming in the form of smart antenna techniques.

However, since in many current wireless communication applications the signal bandwidth is still relatively narrow, almost all of the books within the smart antenna literature are focused on narrowband beamforming and the topic of wideband beamforming is by and large ignored. With the introduction of ultra-wideband systems, one or two chapters on wideband beamforming have recently appeared in books about ultra-wideband communications and wideband radar, etc. With the increasing importance of wideband beamforming and recent advances in this area, it appears timely to have a book dedicated to this topic for the benefit of the wireless communications community. However, the concepts and techniques presented in this book for wideband beamforming are general and not limited to the wireless communications area, or any other specific applications.

There has been a huge amount of work going on in the past half a century in the area of wideband beamforming and it is impossible to cover all of them in the first attempt of producing a single book dedicated to this area. Although we have tried our best to give an extensive review of this topic in Chapters 1, 2 and 4 about both fixed and adaptive beamforming techniques, the remaining part of the book is mostly based on our own research over the past ten years in this area. Our primary goal is to give a systematic introduction to the various concepts and techniques in wideband beamforming in the

form of a self-contained monograph and also present some of the most recent research and development in this area.

The contents of the book are organized into eight chapters.

Chapter 1 is a brief introduction to the general area of array signal processing, including both narrowband and wideband beamforming, with a detailed analysis for the beam steering process for both cases. It will be shown that unlike the narrowband case, where the steered beam response is a circularly shifted version of the original one given a half wavelength spacing, a more complicated relationship exists for a wideband beamformer.

In Chapter 2, we will study a range of basic approaches to adaptive wideband beamforming. The latter can be achieved by a standard adaptive filtering structure when a reference signal is available. When we know the direction of arrival (DOA) angle of the signal of interest, a linearly constrained minimum variance (LCMV) beamformer can be constructed and realized by either a constrained adaptive algorithm or an unconstrained one through the structure of a generalized sidelobe canceller (GSC). In addition to the standard LCMV beamformer, two other minimum variance beamformers will also be studied, including the soft-constrained beamformer and the correlation constrained beamformer. To improve the robustness of the beamformer in the presence of steering vector errors, the topic of robust adaptive beamforming is addressed at the end of this chapter.

Chapter 3 is focused on various subband techniques and structures for adaptive wideband beamforming, which can normally achieve a higher convergence rate and a lower computational complexity. Since the discrete Fourier transform (DFT) and inverse DFT (IDFT) pair can be considered as a simple maximally decimated filter banks system, frequency-domain adaptation techniques are also studied in this chapter.

Chapters 4 and 5 are devoted to the fixed wideband beamformer design problem, with Chapter 4 for a general design using the iterative optimisation method, the least squares method and the eigenfilter method, and Chapter 5 for a special class of fixed wideband beamformers – the frequency invariant beamformer. The design of a frequency invariant beamformer can be achieved by many different methods and at the end of Chapter 5, an application of the frequency invariant beamforming technique to the adaptive wideband beamforming problem is also studied, which leads to a beamspace adaptive wideband beamformer.

Chapter 6 is focused on a different class of adaptive beamformers: the blind adaptive wideband beamformer, which is based on the concept of blind source separation. For this class of beamformers, neither a reference signal nor the DOA information of the desired signal is needed and only some assumptions on the statistical properties of the source signals are required.

In Chapter 7, we will introduce a totally different approach to wideband beamforming based on the recently proposed sensor delay-line system. A special property of the resultant wideband beamforming structure is that there is not any form of temporal processing required, such as tapped delay-lines or FIR/IIR filters. Therefore it can be considered as a wideband beamforming structure with spatial-only information. Most of the techniques developed for the traditional wideband beamformers can be applied to this new structure directly. However, further studies are needed in the future to fully exploit its potential for wideband beamforming in various signal environments and applications.

In the last chapter, Chapter 8, we will study the wideband beamforming problem in a multipath environment. For the case with a small number of multipath signals, two

solutions will be provided employing the wideband beamspace adaptive beamforming structure studied in previous chapters. When a large number of multipath signals is present to the beamformer, we will have a generalized signal mixing problem independent of the array geometry and the original array system can be considered as a general multiple input multiple output (MIMO) system. A brief introduction to the MIMO system is then provided from the viewpoint of beamforming at the end.

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# 1

## Introduction

### 1.1 Array Signal Processing

Array signal processing is one of the major areas of signal processing and has been studied extensively in the past due to its wide applications in various areas ranging from radar, sonar, microphone arrays, radio astronomy, seismology, medical diagnosis and treatment, to communications (Allen and Ghavami, 2005; Brandstein and Ward, 2001; Fourikis, 2000; Haykin, 1985; Hudson, 1981; Johnson and Dudgeon, 1993; Monzingo and Miller, 2004; Van Trees, 2002). It involves multiple sensors (microphones, antennas, etc.) placed at different positions in space to process the received signals arriving from different directions. An example for a simple array system consisting of four sensors with two impinging signals is shown in Figure 1.1 for illustrative purposes, where the direction of arrival (DOA) of the signals is characterized by two parameters: an elevation angle  $\theta$  and an azimuth angle  $\phi$ .

We normally assume the array sensors have the same characteristics and they are omnidirectional (or isotropic), i.e. their responses to an impinging signal are independent of their DOA angles. According to the relative locations of the sensors, arrays can be divided into three classes (Van Trees, 2002):

- one-dimensional (1-D) arrays or linear arrays;
- two-dimensional (2-D) arrays or planar arrays;
- three-dimensional (3-D) arrays or volumetric arrays.

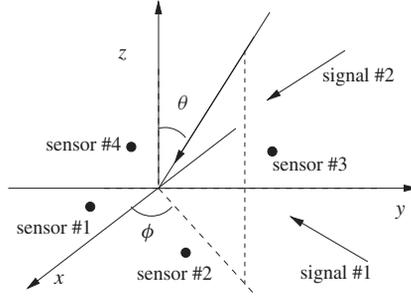
Each of them can be further divided into two categories:

- regular spacing, including uniform and nonuniform spacings;
- irregular or random spacing.

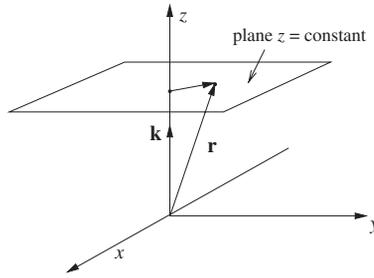
Our study in this book will be based on arrays with regular spacings.

For the impinging signals, we always assume that they are plane waves, i.e. the array is located in the far field of the sources generating the waves and the received signals have a planar wavefront.

Now consider a plane wave with a frequency  $f$  propagating in the direction of the  $z$ -axis of the Cartesian coordinate system as shown in Figure 1.2. At the plane defined



**Figure 1.1** An illustrative array example with four sensors and two impinging signals



**Figure 1.2** A plane wave propagating in the direction of the  $z$ -axis of the Cartesian coordinate system

by  $z = \text{constant}$ , the phase of the signal can be expressed as:

$$\phi(t, z) = 2\pi ft - kz \quad (1.1)$$

where  $t$  is time and the parameter  $k$  is referred to as the wavenumber and defined as (Crawford, 1968):

$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda} \quad (1.2)$$

where  $\omega$  is the (temporal) angular frequency,  $c$  denotes the speed of propagation in the specific medium and  $\lambda$  is the wavelength. Similar to  $\omega$ , which means that in a temporal interval  $t$  the phase of the signal accumulates to the value  $\omega t$ , the interpretation of  $k$  is that over a distance  $z$ , measured in the propagation direction, the phase of the signal accumulates to  $kz$  radians. As a result,  $k$  can be referred to as the spatial frequency of a signal.

Different from the temporal frequency  $\omega$ , which is one-dimensional, the spatial frequency  $k$  is three-dimensional and its direction is opposite to the propagating direction of the signal. In a Cartesian coordinate system, it can be denoted by a three-element vector:

$$\mathbf{k} = [k_x, k_y, k_z]^T \quad (1.3)$$