THEORETICAL PHYSICS

THEORETICAL PHYSICS

BY

W. WILSON, F.R.S.

VOL. I. MECHANICS AND HEAT
NEWTON—CARNOT

VOL. II. ELECTROMAGNETISM AND OPTICS
MAXWELL—LORENTZ

VOL. III. RELATIVITY AND QUANTUM DYNAMICS
EINSTEIN—PLANCK

THEORETICAL PHYSICS

BY

W. WILSON, F.R.S.

HILDRED CARLILE PROFESSOR OF PHYSICS IN THE UNIVERSITY OF LONDON,
BEDFORD COLLEGE

VOL. II

ELECTROMAGNETISM AND OPTICS
MAXWELL—LORENTZ

WITH 78 DIAGRAMS

METHUEN & CO. LTD.
36 ESSEX STREET W.C.
LONDON

First Published in 1933

PRINTED IN GREAT BRITAIN

PREFACE

IN the present volume the exposition of the theory of physics is continued in the same spirit as in the earlier one. The chief consideration which has guided me in selecting the subject-matter has been its importance from the point of view of presenting physical theory as a coherent logical unity. Other considerations have influenced me in a minor degree and I make no claim that my choice of the material is the best that might have been made.

Among the features of this volume, to which attention may be drawn here, are : the form of the electromagnetic field equations on page 114, in which a mere change of notation brings out the 4-dimensional appearance so characteristic of the theory of relativity ; the treatment of electromagnetic momentum and mass in Chapter X, and the subsequent development, in the same chapter, of the field equations in a form which I have called an extended Poisson's equation ; the account of Huygens' principle which is based on the solution of the extended Poisson's equation, and lastly a parallel treatment of electromagnetic momentum and mass, after the manner of H. A. Lorentz, based on the FitzGerald-Lorentz contraction hypothesis. Considerable attention has been given to units and to the dimensions of physical quantities, and the electromagnetic formulae have, with some few exceptions, been developed in such a way that they are valid in *any system of units whatever.*

For convenience of reference the numbering of the sections follows on from that in Vol. I ; but, although many references are made to the earlier volume, the present one is as self-contained as it is possible for a work on electricity and optics to be, when it is remembered that these subjects constitute an organic part of the larger whole of physical science.

All the subject-matter of this volume has been taken from the notes of lectures which I have given at one time or another to university students and, while there is nothing new or original in it, the form in which it is presented has many original features, and I believe that this constitutes the main part of any value the book may have.

I wish to express my indebtedness to Dr. Maud O. Saltmarsh for assisting me in correcting the proofs and to Mr. W. Ewart Williams for permitting me to use two of the figures from his admirable work on Interferometry.

W. W.

August 1933.

CONTENTS

CHAPTER I

CHAPTER II

CHAPTER VII

THERMODYNAMICAL ASPECTS OF ELECTRIC CURRENTS 154

CHAPTER VIII

THE ELECTROMAGNETIC THEORY OF CLERK-MAXWELL 159

CHAPTER IX

SIMPLE ELECTRON THEORY 182

CHAPTER XIII

HUYGENS' PRINCIPLE

CHAPTER XIV

CHAPTER XV

THEORETICAL PHYSICS

CHAPTER I

ELECTROSTATICS

§ 18. ELECTRIFICATION

MANY materials, such as glass, ebonite, resin, etc., are observed, as a consequence of having been rubbed against one another or with other materials, to exert forces on bodies in their neighbourhood. They are said to be **electrified**. A number of glass rods, all of which have been electrified by rubbing with pieces of silk, are found to repel one another; while the pieces of silk likewise repel one another. We cannot distinguish the state of electrification of one glass rod from that of another, nor that of one piece of silk from that of another. So we infer that bodies in the same state of electrification repel one another. But experiment shows that any one of the glass rods attracts any one of the pieces of silk. Consequently the state of electrification of the glass rods differs from that of the silk with which they were rubbed. These two states of electrification were formerly called **vitreous** and **resinous** respectively, since they were observed on glass and resin on rubbing them together. Whenever electrification is produced, both the vitreous and resinous states appear. And when the electrification is due to friction the two bodies which have been rubbed against one another always exhibit different states of electrification.

So far **electrification** is just a name for the state of a body, or of its surface, when it has acquired by friction (or, it may be, by other means) the power to exert mechanical forces on bodies in its neighbourhood; and experiment reveals two types of electrification only.

When a brass rod is rubbed with silk or fur it does not exhibit electrification unless the precaution be taken of mounting it on a support of ebonite, glass or some other material of the class easily electrified by friction, and of holding this support in the hand and not the piece of metal itself. This and similar facts lead us to distinguish between **conductors,** such as the piece of metal, and **insulators** or **dielectrics** like glass, ebonite, sulphur and so on. And there are of course all gradations between almost perfect insulators at one extreme and good conductors at the other.

A conductor, *A*, will exhibit electrification when it is merely brought into the neighbourhood of an electrified body, *B*. This phenomenon is called **induction,** or, more precisely, **electrostatic induction,** to distinguish it from similar phenomena to which the term ‘ induction ’ is also applied. The end of the conductor, *A*, which is next to the electrified body, *B*, always exhibits a state of electrification unlike that on *B*, while the remoter end (if *A* is mounted on an insulating support) exhibits a state of electrification similar to that of *B*.

§ 18·1. QUANTITATIVE ASPECTS OF ELECTRIFICATION

The force between two electrified particles, *not in motion* and situated in an isotropic insulating medium, is directed along the straight line joining them, and the contribution of any one of a number of electrified particles, *A*, *B*, *C*, . . . to the resultant force on another such particle, *X*, is independent of the positions and states of electrification of the remaining particles. That is to say, in calculating the force on *X* we have to assume that the part of it due to *A*, for example, is just the same as it would be if *B*, *C*, . . . were absent. These statements have an experimental basis and we shall adopt them as axioms. Furthermore, the force between two charged particles depends only on the distance between them [1] and, as we shall see, on the insulating medium in which they are situated.

We may now define the meaning of the term **quantity of electricity** or **electric charge** as follows : **The quantity of electricity on an electrified particle, A, is proportional to the force it exerts on a second electrified particle, X, the distance between them and the electrical condition of X remaining constant.** The two particles are of course assumed to be in an infinitely extended isotropic dielectric. It follows at once from the definition just given and the accompanying explanations that the force, **F**, exerted by one charged particle, *A*, on another, *B*, can be expressed in the form :

$$\mathbf{F} = e_1 e_2 \phi(r), \quad . \quad . \quad . \quad . \quad . \quad . \qquad \mathbf{(18{\cdot}1)}$$

---

[1] This is not strictly true for some insulating media which exhibit, in a faint degree, phenomena of hysteresis.

where e_1 and e_2 are the electric charges on the two particles A and B respectively, and $\phi(r)$ is the function which represents the dependence of the force **F** on the distance r between them. When the charges on the two particles are of the same kind, i.e. both of the vitreous kind or both of the resinous kind, the force will tend to separate them; but when they are unlike it will tend to draw them together. When A and B have each the unit charge (which for the present we shall suppose to have been chosen quite arbitrarily) and are separated by the unit distance

$$\mathbf{F} = \phi(1), \quad . \quad . \quad . \quad . \quad . \qquad \textbf{(18·101)}$$

we shall represent the constant $\phi(1)$ by α. Its value will, as we shall see, depend on the insulating medium surrounding the particles.

If a charged conductor be introduced into the interior of an insulated hollow conductor (the aperture in the latter being relatively very small or, better still, closed altogether after the introduction of the charged conductor by the use of a well-fitting conducting lid manipulated by an insulating thread of silk) and then caused to touch the surrounding wall it will be found, on withdrawal and testing with an electroscope, to have lost its charge completely. We shall be able to infer from this fact **(§ 18·4)** that the function $\phi(r)$ has the form

$$\phi(r) = \alpha/r^2. \quad . \quad . \quad . \quad . \quad . \quad . \qquad \textbf{(18·11)}$$

Let us suppose the insulated hollow conductor, which we shall refer to as a Faraday vessel (Faraday's ice-pail), to be connected by a conducting wire to a gold-leaf electroscope, and a charged body to be introduced into it. The leaf of the electroscope will be observed to deflect by a definite amount, which will be quite independent of the position of the charged body in the interior of the Faraday vessel. The deflexion will also be unaffected by the introduction of other bodies (provided they are previously uncharged) and by contact between them and the original electrified body. It will be unaffected when the charged body is made to touch the surrounding Faraday vessel, in which case it will, if it is a conductor, give up the whole of its charge. In short, the deflexion is determined by the charge introduced and is quite independent of its distribution within the enclosure, of the nature of the materials in the enclosure, or of any actions or processes occurring there.

We may use the combination of electroscope and Faraday vessel to measure charges if we provide the electroscope with a scale or use a reading microscope provided with an ocular scale. We should have to calibrate the scale in some such way as the following : We adopt some arbitrary small charge as a unit and note the deflexion produced when it is introduced into the Faraday vessel. Two bodies are now charged with this unit quantity of the same kind of electricity (i.e. both are charged with the vitreous kind of electricity for example) and introduced together into the Faraday vessel. The observed deflexion will represent two units of electricity. We then find the deflexion representing three units of electricity by introducing a body charged with two units and one charged with a single unit and so on.

Let us now suppose that two bodies A and B are charged, the former with *vitreous* and the latter with *resinous* electricity, and that the charges are found to be, for example, 5 and 3 respectively on testing them separately with the measuring device. On introducing A and B *together* into the Faraday vessel a deflexion representing 2 units will be observed, and in fact when A and B are conductors and are brought into contact with the surrounding wall of the vessel, the resulting state of electrification of the Faraday vessel and electroscope is indistinguishable from that due to 2 units of vitreous electricity. Facts such as this have led us to attach the positive sign to the one sort of electricity (vitreous) and the negative sign to the other. There is of course no compelling reason for conferring the favour of the positive sign on vitreous electricity. It might equally well have been assigned to the resinous kind.

We may now, for most purposes, drop the explicit distinction between two sorts of electricity,[1] since it will be taken care of by the sign.

When a glass rod and the silk with which it has been rubbed are introduced together into the Faraday measuring device, no deflexion is observed ; but equal deflexions when they are introduced separately. The net (algebraic) quantity generated by friction is invariably zero. This result is always obtained when charges are generated by friction. This is also true for charges produced inductively, since no change in the deflexion of the electroscope is produced by introducing into the interior of the Faraday vessel an (initially) uncharged conductor and allowing it to hang by the side of a charged body previously introduced. These and similar facts are the basis of a general law of **conservation of electric charge,** according to which we cannot

---

[1] It will force itself on our attention again when we meet with the elementary charges of electricity. The elementary positive charge is associated (in the simplest form in which it is commonly met, the proton) with a much more massive carrier than is the case with the elementary negative charge (electron). This asymmetry is one of the most remarkable facts of physical science.

alter the algebraic or net charge on a body or system of bodies without a compensating change in the charge or charges on bodies external to the system.

We have seen that when a charged conductor is made to touch the interior of a Faraday vessel it parts with its charge entirely. This means in effect that there can be no charge on the interior parts of a conductor. Charges on conductors are confined to their surfaces. This statement can be generalized as follows: **The algebraic sum of the charges within a closed conducting surface is always zero.** If for example we introduce a charged conductor into our Faraday vessel and then touch the latter, or in some way connect it conductively to the earth, the deflexion of the electroscope will drop to zero, and the Faraday vessel with its contents will behave, so far as exterior bodies are concerned, as if it were devoid of charges. If the system be insulated once again and the charged body removed, without having been in contact with the surrounding Faraday vessel, the original deflexion will be reproduced, showing the presence of the induced charge which just sufficed to make the algebraic sum of the charges in the interior equal to zero.

A closed conducting surface completely screens, as we have seen, the region exterior to it from the field within, and no change whatever that may be made in the disposition of the charges in the interior, or the character of the **electrostatic field** there, will be observable outside[1]—always provided, of course, that the change does not include the introduction or removal of charges. Experiments have also been carried out with electroscopes and other electrostatic apparatus by observers situated *within* closed conducting surfaces, or within regions which may be regarded as bounded by such surfaces, e.g. within wire cages. These have demonstrated that the conducting surface completely screens the *interior* region from an external electrostatic field.

When the *conductors* in an electrostatic field are replaced by others of different composition, the shapes of the conductors, their positions and charges remaining unchanged, the observable electrostatic phenomena are not in any way affected.

§ 18·2. ELECTROSTATIC FIELDS. POTENTIAL

An electrostatic field is completely described when we have given, for every point in it, the magnitude and direction of the

---

[1] Observable by any electrostatic measuring devices. We might indeed detect a *flow* of electricity in the interior, since one of its consequences would be a rise in temperature which would be observable *outside*.

force which would be exerted on a unit positive charge placed there, or rather, if we express it with strict accuracy, **the quotient of the force exerted on a particle with an infinitesimal charge (and finite charge density) by the amount of the charge taken with its proper sign.** This force per unit charge is termed the **field intensity (§ 2·4)** and we shall represent it by

$$\boldsymbol{\mathcal{E}} \equiv (\mathcal{E}_x, \mathcal{E}_y, \mathcal{E}_z). \quad . \quad . \quad . \quad . \qquad \mathbf{(18{\cdot}2)}$$

Attention should be paid to the way in which the definition is framed. The charged particle is to be regarded as a charged body for which the quotient, charge over volume, is finite, and the quantity which is adopted as a measure of the field intensity is the limiting value of the quotient, force over charge, when the volume of the particle becomes infinitesimal. It will be seen that, with this definition as a basis, any method adopted to measure field intensity will not do violence to the character of the field being measured.

Turning to the simple case of the field due to a single particle with the charge e, we find as a consequence of the definition of field intensity and equation **(18·1)**, that

$$\boldsymbol{\mathcal{E}} = e\phi(r) \quad . \quad . \quad . \quad . \quad . \quad . \qquad \mathbf{(18{\cdot}21)}$$

at a point distant r from the charged particle ; and if, in order to facilitate descriptions, we suppose the particle to be at the origin of rectangular co-ordinates and regard r as a vector,

$$\mathbf{r} \equiv (x, y, z), \quad . \quad . \quad . \quad . \quad . \qquad \mathbf{(28{\cdot}22)}$$

the vector $\boldsymbol{\mathcal{E}}$ will have the same direction as **(18·22)** when the charge e is positive. Therefore

$$\mathcal{E}_x/\boldsymbol{\mathcal{E}} = x/\mathbf{r},\ \mathcal{E}_y/\boldsymbol{\mathcal{E}} = y/\mathbf{r},\ \mathcal{E}_z/\boldsymbol{\mathcal{E}} = z/\mathbf{r}.$$

Consequently

$$\left.\begin{aligned} \mathcal{E}_x &= e\phi(r)\frac{x}{r} = e\phi(r)\frac{\partial r}{\partial x}, \\ \mathcal{E}_y &= e\phi(r)\frac{y}{r} = e\phi(r)\frac{\partial r}{\partial y}, \\ \mathcal{E}_z &= e\phi(r)\frac{z}{r} = e\phi(r)\frac{\partial r}{\partial z}. \end{aligned}\right\} \quad . \quad . \qquad \mathbf{(18{\cdot}221)}$$

Let $\mathbf{dl} \equiv (dx, dy, dz)$ represent a small displacement. The work done on a particle, with a small charge on it, when it is