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Option theory

PETER JAMES

Option Theory

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Riccardo Rebonato

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To Vivien

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Preface

Options are financial instruments which are bought and sold in a market place. The people who do it well pocket large bonuses; companies that do it badly can suffer staggering losses. These are intensely practical activities and this is a technical book for practical people working in the industry. While writing it I have tried to keep a number of issues and principles to the forefront:

- The emphasis is on developing the theory to the point where it is capable of yielding a numerical answer to a pricing question, either through a formula or through a numerical procedure. In those places where the theory is fairly abstract, as in the sections explaining stochastic calculus, the path back to reality is clearly marked.
- An objective of the book is to demystify option theory. An essential part of this is giving explanations and derivations in full. I have (almost) completely avoided the "it can be shown that . . . " syndrome, except for the most routine algebraic steps, since this can be very time-wasting and frustrating for the reader. No quant who values his future is going to just lift a formula or set of procedures from a textbook and apply them without understanding where they came from and what assumptions went into them.
- It is a sad fact that readers do not start at the beginning of a textbook and read every page until they get to the end – at least not the people I meet in the derivatives market. Practitioners are usually looking for something specific and want it quickly. I have therefore tried to make the book reasonably easy to dip in and out of. This inevitably means a little duplication and a lot of signposts to parts of the book where underlying principles are explained.
- Option theory can be approached from several different directions, using different mathematical techniques. An option price can be worked out by solving a differential equation or by taking a risk-neutral expectation; results can be obtained by using formulas or trees or by integrating numerically or by using finite difference methods; and the theoretical underpinnings of option theory can be explained either by using conventional, classical statistical methods or by using axiomatic probability theory and stochastic calculus. This book demonstrates that these are all saying the same thing in different languages; there is only one option theory, although several branches of mathematics can be used to describe it. I have taken pains to be unpartisan in describing techniques; the best technique is the one that produces the best answer, and this is not the same for all options.

The reader of this book might have no previous knowledge of option theory at all, or he might be an accomplished quant checking an obscure point. He might be a student looking to complement his course material or he might be a practitioner who wants to understand the use of stochastic calculus in option theory; but he will start with an intermediate knowledge of calculus and the elements of statistics. The book is divided into four parts and a substantial mathematical appendix. The first three parts cover (1) the basic principles of option theory, (2) computational methods and (3) the application of the previous theory to exotic options. The mathematical tools needed for these first three parts are pre-packaged in the appendix, in a consistent form that can be used with minimal interruption to the flow of the text.

Part 4 has the ambitious objective of giving the reader a working knowledge of stochastic calculus. A pure mathematician's approach to this subject would start with a heavy dose of measure theory and axiomatic probability theory. This is an effective barrier to entry for many students and practitioners. Furthermore, as with any restricted trade, those who have crossed the barrier have every interest in making sure that it stays in place: who needs extra competition for those jobs or consulting contracts? This has unfortunately led to many books and articles being unnecessarily dressed up in stochastic jargon; at the same time there are many students and practitioners with perfectly adequate freshman level calculus and statistics who are frustrated by their inability to penetrate the literature.

This particular syndrome has been sorted out in mature fields such as engineering and science. If you want to be a pure mathematician, you devote your studies to the demanding questions of pure mathematics. If you want to be an engineer, you still need a lot of mathematics, but you will learn it from books with titles such as "Advanced Engineering Mathematics". Nobody feels there is much value in turning electrical engineering or solid state physics into a playground for pure mathematicians.

It is assumed that before embarking on Part 4, the reader will already have a rudimentary knowledge of option theory. He may be shaky on detail, but he will know how a risk-free portfolio leads to the risk-neutrality concept and how a binomial tree works. At this point he already knows quite a lot of useful stochastic theory without realizing it and without knowing the fancy words. This knowledge can be built upon and developed into discrete stochastic theory using familiar concepts. In the limit of small time steps this generalizes to a continuous stochastic theory; the generalization is not always smooth and easy, but anomalies created by the transition are explicitly pointed out. A completely rigorous approach would lead us through an endless sea of lemmas, so we take the engineer's way. Our ultimate interest is in option theory, so frequent recourse is made to heuristic or intuitive reasoning. We do so without apology, for a firm grasp of the underlying "physical" processes ultimately leads to a sounder understanding of derivatives than an over-reliance on abstract mathematical manipulation.

The objective is to give the reader a sufficient grasp of stochastic calculus to allow him to understand the literature and use it actively. There is little benefit to the reader in a dumbed down sketch of stochastic theory which still leaves him unable to follow the serious literature. The necessary jargon is therefore described and the theory is developed with constant reference to option theory. By the end of Part 4 the attentive reader will have a working knowledge of martingales, stochastic differential equations and integration, the Feynman Kac theorem, local time, stochastic control and Girsanov's theorem.

A final chapter in Part 4 applies all these tools to various problems encountered in studying equity-type derivatives. Some of these problems had been encountered earlier in the book and are now solved more gracefully; others are really not convincingly soluble without stochastic calculus. Of course the most important application in this latter category is the whole subject of interest rate derivatives. However, the book stops short at this point for two reasons: first, the field of derivatives has now become so large that it is no longer feasible to cover both equity and interest rate options thoroughly in a single book of reasonable length. Second, three or four very similar texts on this subject have appeared in the last couple of years; they are all quite good and they all launch into interest rate derivatives at the point where this book finishes. Any reader primarily motivated by an interest in interest rate options, but floundering in stochastic calculus, will find Part 4 a painless way into these more specialist texts.

Peter James
option.theory@james-london.com

Part 1
Elements of Option Theory

1 Fundamentals

The trouble with first chapters is that nearly everyone jumps over them and goes straight to the meat. So, assuming the reader gets this far before jumping, let me say what will be missed and why it might be worth coming back sometime.

Section 1.1 is truly jumpable, so long as you really understand continuous as opposed to discrete interest and dividends, sign conventions for long and short securities positions and conventions for designating the passing of time. Section 1.2 gives a first description of the concept of arbitrage, which is of course central to the subject of this book. This description is rather robust and intuitive, as opposed to the fancy definition couched in heavy mathematics which is given much later in the book; it is a practical working-man's view of arbitrage, but it yields most of the results of modern option theory.

Forward contracts are really only common in the foreign exchange markets; but the concept of a forward rate is embedded within the analysis of more complex derivatives such as options, in all financial markets. We look at forward contracts in Section 1.3 and introduce one of the central mysteries of option theory: risk neutrality.

Finally, Section 1.4 gives a brief description of the nature of a futures contract and its relationship with a forward contract.

1.1 CONVENTIONS

(i) ***Continuous Interest*:** If we invest \$100 for a year at an annual rate of 10%, we get \$110 after a year; at a semi-annual rate of 10%, we get $\$100 \times 1.05^2 = \$110.25$ after a year, and at a quarterly rate, $\$100 \times 1.025^4 = \$110.38$. In the limit, if the interest is compounded each second, we get

$$\$100 \times \lim_{n\to\infty}\left(1 + \frac{0.1}{n}\right)^n = \$100 \times \mathrm{e}^{0.1} = \$110.52$$

The factor by which the principal sum is multiplied when we have continuous compounding is $\mathrm{e}^{r_\mathrm{c} T}$, where T is the time to maturity and r_c is the continuously compounding rate.

In commercial contracts, interest payments are usually specified with a stated compounding period, but in option theory we always use continuous compounding for two reasons: first, the exponential function is analytically simpler to handle; and second, the compounding period does not have to be specified.

When actual rates quoted in the market need to be used, it is a simple matter to convert between continuous and discrete rates:

$$\text{Annual Compounding:}\quad \mathrm{e}^{r_\mathrm{c}} = 1 + r_1 \quad\Rightarrow r_\mathrm{c} = \ln(1 + r_1)$$

$$\text{Semi-annual Compounding:}\quad \mathrm{e}^{r_\mathrm{c}} = \left(1 + \frac{r_{1/2}}{2}\right)^2 \Rightarrow r_\mathrm{c} = 2\ \ln\left(1 + \frac{r_{1/2}}{2}\right)$$

$$\text{Quarterly Compounding:}\quad \mathrm{e}^{r_\mathrm{c}} = \left(1 + \frac{r_{1/4}}{4}\right)^4 \Rightarrow r_\mathrm{c} = 4\ \ln\left(1 + \frac{r_{1/4}}{4}\right)$$