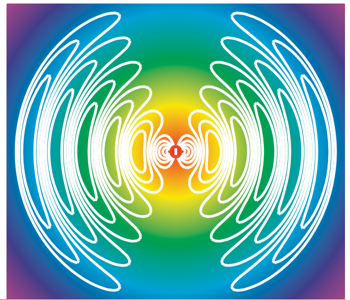
Electromagnetic Waves and Antennas

Sophocles J. Orfanidis



Maxwell's Equations
Uniform Plane Waves
Pulse Propagation in Dispersive Media
Slow, Fast, Negative Group Velocity
Dispersion Compensation
Pulse Compression and Chirp Radar
Propagation in Birefringent Media
Reflection and Transmission
Multilayer Dielectric Structures
Oblique Incidence
Negative-Index Media
Perfect Lenses
Multilayer Film Applications
Surface Plasmon Resonance
Omnidirectional Dielectric Mirrors
Giant Birefringent Optics
Waveguides
Transmission Lines
Coupled Lines
Impedance Matching
S-Parameters
Microwave Amplifier Design
Radiation Fields
Transmitting and Receiving Antennas
Linear and Loop Antennas
Radiation from Apertures
Fresnel Diffraction
Plane Wave Spectrum Representation
Fourier Optics
Aperture Antennas
Horn and Patch Antennas
Parabolic Antennas
Antenna Arrays
Array Design Methods
Currents on Linear Antennas
Hallen and Pocklington Equations
Coupled Antennas

MATLAB Functions

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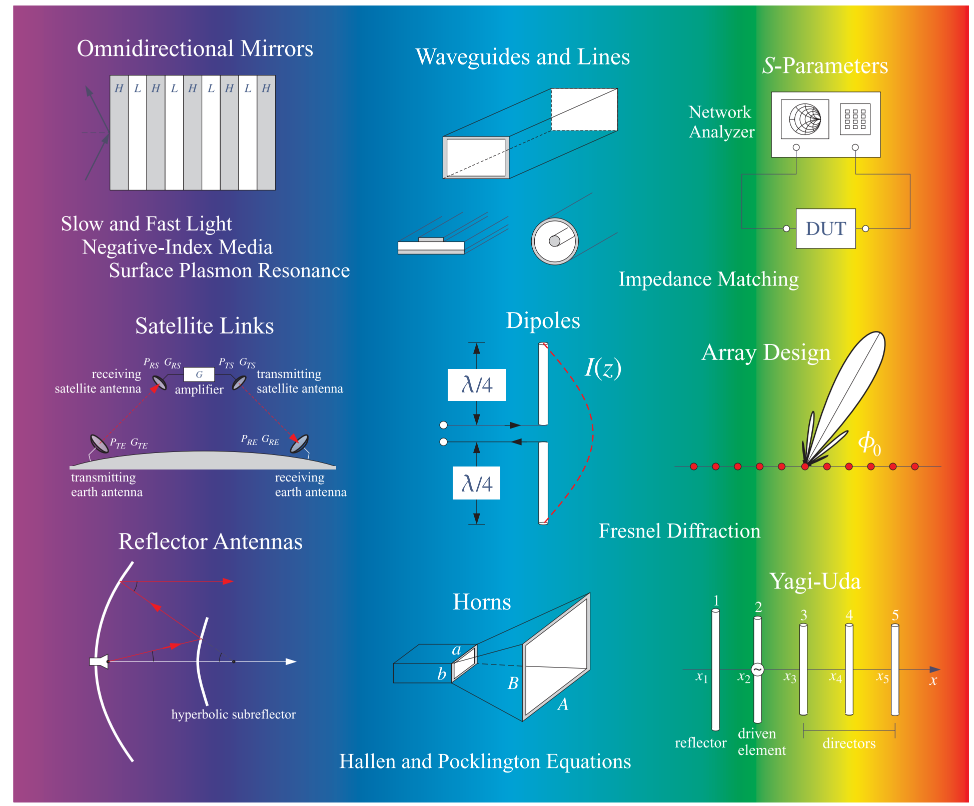

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Sophocles J. Orfanidis

Rutgers University

To Monica and John

Web page: `www.ece.rutgers.edu/~orfanidi/ewa`

Contents

Preface

This text provides a broad and applications-oriented introduction to electromagnetic waves and antennas. Current interest in these areas is driven by the growth in wireless and fiber-optic communications, information technology, and materials science.

Communications, antenna, radar, and microwave engineers must deal with the generation, transmission, and reception of electromagnetic waves. Device engineers working on ever-smaller integrated circuits and at ever higher frequencies must take into account wave propagation effects at the chip and circuit-board levels. Communication and computer network engineers routinely use waveguiding systems, such as transmission lines and optical fibers. Novel recent developments in materials, such as photonic bandgap structures, omnidirectional dielectric mirrors, birefringent multilayer films, surface plasmons, negative-index metamaterials, slow and fast light, promise a revolution in the control and manipulation of light and other applications. These are just some examples of topics discussed in this book. The text is organized around three main topic areas:

- The propagation, reflection, and transmission of plane waves, and the analysis and design of multilayer films.
- Waveguides, transmission lines, impedance matching, and *S*-parameters.
- Linear and aperture antennas, scalar and vector diffraction theory, antenna array design, numerical methods in antennas, and coupled antennas.

The text emphasizes connections to other subjects. For example, the mathematical techniques for analyzing wave propagation in multilayer structures and the design of multilayer optical filters are the same as those used in digital signal processing, such as the lattice structures of linear prediction, the analysis and synthesis of speech, and geophysical signal processing. Similarly, antenna array design is related to the problem of spectral analysis of sinusoids and to digital filter design, and Butler beams are equivalent to the FFT.

Use

The book is appropriate for first-year graduate or senior undergraduate students. There is enough material in the book for a two-semester course sequence. The book can also be used by practicing engineers and scientists who want a quick review that covers most of the basic concepts and includes many application examples.

The book is based on lecture notes for a first-year graduate course on "Electromagnetic Waves and Radiation" that I have been teaching at Rutgers for more than twenty years. The course draws students from a variety of fields, such as solid-state devices, wireless communications, fiber optics, biomedical engineering, and digital signal and array processing. Undergraduate seniors have also attended the graduate course successfully.

The book requires a prerequisite course on electromagnetics, typically offered at the junior year. Such introductory course is usually followed by a senior-level elective course on electromagnetic waves, which covers propagation, reflection, and transmission of waves, waveguides, transmission lines, and perhaps some antennas. This book may be used in such elective courses with the appropriate selection of chapters.

At the graduate level, there is usually an introductory course that covers waves, guides, lines, and antennas, and this is followed by more specialized courses on antenna design, microwave systems and devices, optical fibers, and numerical techniques in electromagnetics. No single book can possibly cover all of the advanced courses. This book may be used as a text in the initial course, and as a supplementary text in the specialized courses.

Contents and Highlights

The first eight chapters develop waves concepts and applications. The material progresses from Maxwell equations, to uniform plane waves in various media, such as lossless and lossy dielectrics and conductors, birefringent and chiral media, including negative-index media, to reflection and transmission problems at normal and oblique incidence, including reflection from moving boundaries and the Doppler effect, to multilayer structures.

Chapter three deals with pulse propagation in dispersive media, with discussions of group and front velocity and causality, group velocity dispersion, spreading and chirping, dispersion compensation, slow, fast, and negative group velocity, and an introduction to chirp radar and pulse compression.

Some of the oblique incidence applications include inhomogeneous waves, total internal reflection, surface plasmons, ray tracing and atmospheric refraction, and Snel's law in negative-index media.

The material on multilayer structures includes the design of antireflection coatings, omnidirectional dielectric mirrors, broadband reflectionless multilayers, frustrated total internal reflection and surface plasmon resonance, perfect lenses in negative-index media, polarizing beam splitters, and birefringent multilayer structures.

Chapters 9–13 deal with waveguides and transmission lines. We cover only rectangular waveguides, resonant cavities, and simple dielectric waveguides. The transmission line material includes a discussion of microstrip and coaxial lines, terminated lines, standing wave ratio and the Smith chart, and examples of time-domain transient response of lines. We have included some material on coupled lines and crosstalk, as well as some on coupled mode theory and fiber Bragg gratings.

We devote one chapter to impedance matching methods, including multisection Chebyshev quarter-wavelength transformers, quarter-wavelength transformers with series or shunt stubs, single stub tuners, as well as L-section and Π-section reactive matching networks.

Chapter 13 presents an introduction to S-parameters with a discussion of input and output reflection coefficients, two-port stability conditions, transducer, operating, and available power gains, power waves, simultaneous conjugate matching, noise figure circles, illustrating the concepts with a number of low-noise high-gain microwave amplifier designs including the design of their input and output matching circuits.

Chapters 14–22 deal with radiation and antenna concepts. We begin by deriving expressions for the radiation fields from current sources, including magnetic currents, and then apply them to linear and aperture antennas. Chapter 15 covers general fundamental antenna concepts, such as radiation intensity, power density, directivity and gain, beamwidth, effective area, effective length, Friis formula, antenna noise temperature, power budgets in satellite links, and the radar equation.

We have included a number of linear antenna examples, such as Hertzian and half-wave dipoles, traveling, vee, and rhombic antennas, as well as loop antennas.

Two chapters are devoted to radiation from apertures. The first discusses Schelkunoff's field equivalence principle, magnetic currents and duality, radiation fields from apertures, vector diffraction theory, including the Kottler, Stratton-Chu, and Franz formulations, extinction theorem, Fresnel diffraction, Fresnel, zones, Sommerfeld's solution to the knife-edge diffraction problem, geometrical theory of diffraction, Rayleigh-Sommerfeld diffraction theory and its connection to the plane-wave spectrum representation with applications to Fourier optics.

The second presents a number of aperture antenna examples, such as open-ended waveguides, horn antennas, including optimum horn design, microstrip antennas, parabolic and dual reflectors, and lens antennas.

Two other chapters discuss antenna arrays. The first introduces basic concepts such as the multiplicative array pattern, visible region, grating lobes, directivity including its optimization, array steering, and beamwidth.

The other discusses several array design methods, such as by zero placement, Fourier series method with windowing, sector beam design, Woodward-Lawson method, and several narrow-beam low-sidelobe designs, such as binomial, Dolph-Chebyshev, Taylor's one-parameter, Taylor's $\bar{n}$ distribution, prolate, and Villeneuve array design. We have expanded on the analogies with time-domain DSP concepts and filter design methods. We finally give some examples of multibeam designs, such as Butler beams.

The last two chapters deal with numerical methods for linear antennas. Chapter 21 develops the Hallén and Pocklington integral equations for determining the current on a linear antenna, discusses King's three-term approximations, and then concentrates on numerical solutions for delta-gap input and arbitrary incident fields. We discuss the method of moments, implemented with the exact or the approximate thin-wire kernel and using various bases, such as pulse, triangular, and NEC bases. These methods require the accurate evaluation of the exact thin-wire kernel, which we approach using an elliptic function representation.

In Chapter 22 we discuss coupled antennas, in particular, parallel dipoles. Initially, we assume sinusoidal currents and reduce the problem to the calculation of the mutual impedance matrix. Then, we consider a more general formulation that requires the solution of a system of coupled Hallén equations. We present various examples, including the design of Yagi-Uda antennas.

Our MATLAB-based numerical solutions are not meant to replace sophisticated commercial field solvers. The inclusion of numerical methods in this book was motivated by the desire to provide the reader with some simple tools for self-study and experimentation. The study of numerical methods in electromagnetics is a subject in itself and our treatment does not do justice to it. However, we felt that it would be fun to be able to quickly compute fairly accurate radiation patterns in various antenna examples, such as Yagi-Uda and other coupled antennas, as well horns and reflector antennas.

The appendix includes summaries of physical constants, electromagnetic frequency bands, vector identities, integral theorems, Green's functions, coordinate systems, Fresnel integrals, sine and cosine integrals, the stationary phase approximation, Gauss-Legendre quadrature, Lorentz transformations, and a detailed list of the MATLAB functions.

Finally, there is a large (but inevitably incomplete) list of references, arranged by topic area, as well as several web links, that we hope could serve as a starting point for further study.

MATLAB Toolbox

The text makes extensive use of MATLAB. We have developed an "Electromagnetic Waves & Antennas" toolbox containing 170 MATLAB functions for carrying out all of the computations and simulation examples in the text. Code segments illustrating the usage of these functions are found throughout the book, and serve as a user manual. The functions may be grouped into the following categories:

1. Design and analysis of multilayer film structures, including antireflection coatings, polarizers, omnidirectional mirrors, narrow-band transmission filters, surface plasmon resonance, birefringent multilayer films and giant birefringent optics.
2. Design of quarter-wavelength impedance transformers and other impedance matching methods, such as Chebyshev transformers, dual-band transformers, stub matching and L-, Π- and T-section reactive matching networks.
3. Design and analysis of transmission lines and waveguides, such as microstrip lines and dielectric slab guides.
4. S-parameter functions for gain computations, Smith chart generation, stability, gain, and noise-figure circles, simultaneous conjugate matching, and microwave amplifier design.
5. Functions for the computation of directivities and gain patterns of linear antennas, such as dipole, vee, rhombic, and traveling-wave antennas, including functions for the input impedance of dipoles.
6. Aperture antenna functions for open-ended waveguides, horn antenna design, diffraction integrals, and knife-edge diffraction coefficients.
7. Antenna array design functions for uniform, binomial, Dolph-Chebyshev, Taylor one-parameter, Taylor $\bar{n}$ distribution, prolate, Villeneuve arrays, sector-beam,

multi-beam, Woodward-Lawson, and Butler beams. Functions for beamwidth and directivity calculations, and for steering and scanning arrays.

8. Numerical methods for solving the Hallén and Pocklington integral equations for single and coupled antennas, computing the exact thin-wire kernel, and computing self and mutual impedances.
9. Several functions for making azimuthal and polar plots of antenna and array gain patterns in decibels and absolute units.
10. There are also several MATLAB movies showing pulse propagation in dispersive media illustrating slow, fast, and negative group velocity; the propagation of step signals and pulses on terminated transmission lines; the propagation on cascaded lines; step signals getting reflected from reactive terminations; fault location by TDR; crosstalk signals propagating on coupled lines; and the time-evolution of the field lines radiated by a Hertzian dipole.

The MATLAB functions as well as other information about the book may be downloaded from the web page: `www.ece.rutgers.edu/~orfanidi/ewa`.

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1

Maxwell's Equations

1.1 Maxwell's Equations

Maxwell's equations describe all (classical) electromagnetic phenomena:

$$\boxed{\begin{aligned} \nabla\times\boldsymbol{E} &= -\frac{\partial\boldsymbol{B}}{\partial t}\\ \nabla\times\boldsymbol{H} &= \boldsymbol{J}+\frac{\partial\boldsymbol{D}}{\partial t}\\ \nabla\cdot\boldsymbol{D} &= \rho\\ \nabla\cdot\boldsymbol{B} &= 0\end{aligned}} \qquad \text{(Maxwell's equations)} \tag{1.1.1}$$

The first is *Faraday's law of induction*, the second is *Ampère's law* as amended by Maxwell to include the displacement current $\partial\boldsymbol{D}/\partial t$, the third and fourth are *Gauss' laws* for the electric and magnetic fields.

The displacement current term $\partial\boldsymbol{D}/\partial t$ in Ampère's law is essential in predicting the existence of propagating electromagnetic waves. Its role in establishing charge conservation is discussed in Sec. 1.6.

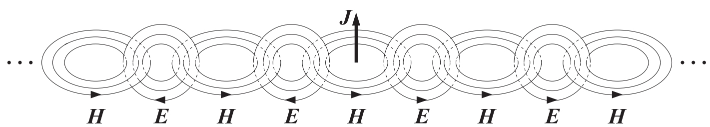
Eqs. (1.1.1) are in SI units. The quantities $\boldsymbol{E}$ and $\boldsymbol{H}$ are the electric and magnetic *field intensities* and are measured in units of [volt/m] and [ampere/m], respectively. The quantities $\boldsymbol{D}$ and $\boldsymbol{B}$ are the electric and magnetic *flux densities* and are in units of [coulomb/m^2] and [weber/m^2], or [tesla]. $\boldsymbol{B}$ is also called the *magnetic induction.*

The quantities ρ and $\boldsymbol{J}$ are the *volume charge density* and *electric current density* (charge flux) of any *external* charges (that is, not including any induced polarization charges and currents.) They are measured in units of [coulomb/m^3] and [ampere/m^2]. The right-hand side of the fourth equation is zero because there are no magnetic monopole charges. Eqs. (1.3.17)–(1.3.19) display the induced polarization terms explicitly.

The charge and current densities $\rho, \boldsymbol{J}$ may be thought of as the *sources* of the electromagnetic fields. For wave propagation problems, these densities are localized in space; for example, they are restricted to flow on an antenna. The generated electric and magnetic fields are *radiated* away from these sources and can propagate to large distances to the receiving antennas. Away from the sources, that is, in source-free regions of space, Maxwell's equations take the simpler form:

$$\boxed{\begin{aligned}\nabla\times\boldsymbol{E} &= -\frac{\partial\boldsymbol{B}}{\partial t}\\ \nabla\times\boldsymbol{H} &= \frac{\partial\boldsymbol{D}}{\partial t}\\ \nabla\cdot\boldsymbol{D} &= 0\\ \nabla\cdot\boldsymbol{B} &= 0\end{aligned}}\qquad\text{(source-free Maxwell's equations)}\tag{1.1.2}$$

The qualitative mechanism by which Maxwell's equations give rise to propagating electromagnetic fields is shown in the figure below.



For example, a time-varying current $\boldsymbol{J}$ on a linear antenna generates a circulating and time-varying magnetic field $\boldsymbol{H}$, which through Faraday's law generates a circulating electric field $\boldsymbol{E}$, which through Ampère's law generates a magnetic field, and so on. The cross-linked electric and magnetic fields propagate away from the current source. A more precise discussion of the fields radiated by a localized current distribution is given in Chap. 14.

1.2 Lorentz Force

The force on a charge q moving with velocity $\boldsymbol{v}$ in the presence of an electric and magnetic field $\boldsymbol{E},\boldsymbol{B}$ is called the Lorentz force and is given by:

$$\boldsymbol{F} = q\,(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})\qquad\text{(Lorentz force)}\tag{1.2.1}$$

Newton's equation of motion is (for non-relativistic speeds):

$$m\,\frac{d\boldsymbol{v}}{dt} = \boldsymbol{F} = q\,(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})\tag{1.2.2}$$

where m is the mass of the charge. The force $\boldsymbol{F}$ will increase the kinetic energy of the charge at a rate that is equal to the rate of work done by the Lorentz force on the charge, that is, $\boldsymbol{v}\cdot\boldsymbol{F}$. Indeed, the time-derivative of the kinetic energy is:

$$W_{\text{kin}} = \frac{1}{2}m\,\boldsymbol{v}\cdot\boldsymbol{v}\quad\Rightarrow\quad \frac{dW_{\text{kin}}}{dt} = m\,\boldsymbol{v}\cdot\frac{d\boldsymbol{v}}{dt} = \boldsymbol{v}\cdot\boldsymbol{F} = q\,\boldsymbol{v}\cdot\boldsymbol{E}\tag{1.2.3}$$

We note that only the electric force contributes to the increase of the kinetic energy—the magnetic force remains perpendicular to $\boldsymbol{v}$, that is, $\boldsymbol{v}\cdot(\boldsymbol{v}\times\boldsymbol{B}) = 0$.

Volume charge and current distributions $\rho,\boldsymbol{J}$ are also subjected to forces in the presence of fields. The Lorentz force *per unit volume* acting on $\rho,\boldsymbol{J}$ is given by:

$$\boldsymbol{f} = \rho\boldsymbol{E} + \boldsymbol{J}\times\boldsymbol{B}\qquad\text{(Lorentz force per unit volume)}\tag{1.2.4}$$

where $\boldsymbol{f}$ is measured in units of [newton/m^3]. If $\boldsymbol{J}$ arises from the motion of charges within the distribution ρ, then $\boldsymbol{J} = \rho\boldsymbol{v}$ (as explained in Sec. 1.5.) In this case,

$$\boldsymbol{f} = \rho\,(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})\tag{1.2.5}$$

By analogy with Eq. (1.2.3), the quantity $\boldsymbol{v}\cdot\boldsymbol{f} = \rho\,\boldsymbol{v}\cdot\boldsymbol{E} = \boldsymbol{J}\cdot\boldsymbol{E}$ represents the *power per unit volume* of the forces acting on the moving charges, that is, the power expended by (or lost from) the fields and converted into kinetic energy of the charges, or heat. It has units of [watts/m^3]. We will denote it by:

$$\boxed{\frac{dP_{\text{loss}}}{dV} = \boldsymbol{J}\cdot\boldsymbol{E}}\qquad\text{(ohmic power losses per unit volume)}\tag{1.2.6}$$

In Sec. 1.7, we discuss its role in the conservation of energy. We will find that electromagnetic energy flowing into a region will partially increase the stored energy in that region and partially dissipate into heat according to Eq. (1.2.6).

1.3 Constitutive Relations

The electric and magnetic flux densities $\boldsymbol{D},\boldsymbol{B}$ are related to the field intensities $\boldsymbol{E},\boldsymbol{H}$ via the so-called *constitutive relations*, whose precise form depends on the material in which the fields exist. In *vacuum*, they take their simplest form:

$$\boxed{\begin{aligned}\boldsymbol{D} &= \epsilon_0\boldsymbol{E}\\ \boldsymbol{B} &= \mu_0\boldsymbol{H}\end{aligned}}\tag{1.3.1}$$

where ϵ_0,μ_0 are the *permittivity* and *permeability* of vacuum, with numerical values:

$$\boxed{\begin{aligned}\epsilon_0 &= 8.854\times10^{-12}\ \text{farad/m}\\ \mu_0 &= 4\pi\times10^{-7}\ \text{henry/m}\end{aligned}}\tag{1.3.2}$$

The units for ϵ_0 and μ_0 are the units of the ratios D/E and B/H, that is,

$$\frac{\text{coulomb/m}^2}{\text{volt/m}} = \frac{\text{coulomb}}{\text{volt}\cdot\text{m}} = \frac{\text{farad}}{\text{m}},\qquad \frac{\text{weber/m}^2}{\text{ampere/m}} = \frac{\text{weber}}{\text{ampere}\cdot\text{m}} = \frac{\text{henry}}{\text{m}}$$

From the two quantities ϵ_0,μ_0, we can define two other physical constants, namely, the *speed of light* and *characteristic impedance* of vacuum:

$$\boxed{c_0 = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 3\times10^8\ \text{m/sec}\,,\qquad \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\ \text{ohm}}\tag{1.3.3}$$

The next simplest form of the constitutive relations is for simple homogeneous isotropic dielectric and for magnetic materials:

$$\boxed{\begin{aligned} \boldsymbol{D} &= \epsilon \boldsymbol{E} \\ \boldsymbol{B} &= \mu \boldsymbol{H} \end{aligned}} \tag{1.3.4}$$

These are typically valid at low frequencies. The permittivity ϵ and permeability μ are related to the *electric and magnetic susceptibilities* of the material as follows:

$$\boxed{\begin{aligned} \epsilon &= \epsilon_0(1+\chi) \\ \mu &= \mu_0(1+\chi_m) \end{aligned}} \tag{1.3.5}$$

The susceptibilities χ, χ_m are measures of the electric and magnetic polarization properties of the material. For example, we have for the electric flux density:

$$\boldsymbol{D} = \epsilon \boldsymbol{E} = \epsilon_0(1+\chi)\boldsymbol{E} = \epsilon_0 \boldsymbol{E} + \epsilon_0 \chi \boldsymbol{E} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P} \tag{1.3.6}$$

where the quantity $\boldsymbol{P} = \epsilon_0 \chi \boldsymbol{E}$ represents the dielectric polarization of the material, that is, the average electric dipole moment per unit volume. In a magnetic material, we have

$$\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M}) = \mu_0(\boldsymbol{H} + \chi_m \boldsymbol{H}) = \mu_0(1+\chi_m)\boldsymbol{H} = \mu \boldsymbol{H} \tag{1.3.7}$$

where $\boldsymbol{M} = \chi_m \boldsymbol{H}$ is the *magnetization*, that is, the average magnetic moment per unit volume. The speed of light in the material and the characteristic impedance are:

$$c = \frac{1}{\sqrt{\mu\epsilon}}, \qquad \eta = \sqrt{\frac{\mu}{\epsilon}} \tag{1.3.8}$$

The *relative permittivity, permeability* and *refractive index* of a material are defined by:

$$\epsilon_{\text{rel}} = \frac{\epsilon}{\epsilon_0} = 1 + \chi, \qquad \mu_{\text{rel}} = \frac{\mu}{\mu_0} = 1 + \chi_m, \qquad n = \sqrt{\epsilon_{\text{rel}}\mu_{\text{rel}}} \tag{1.3.9}$$

so that $n^2 = \epsilon_{\text{rel}}\mu_{\text{rel}}$. Using the definition of Eq. (1.3.8), we may relate the speed of light and impedance of the material to the corresponding vacuum values:

$$\begin{aligned} c &= \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\epsilon_0\epsilon_{\text{rel}}\mu_{\text{rel}}}} = \frac{c_0}{\sqrt{\epsilon_{\text{rel}}\mu_{\text{rel}}}} = \frac{c_0}{n} \\ \eta &= \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0}}\sqrt{\frac{\mu_{\text{rel}}}{\epsilon_{\text{rel}}}} = \eta_0\sqrt{\frac{\mu_{\text{rel}}}{\epsilon_{\text{rel}}}} = \eta_0\,\frac{\mu_{\text{rel}}}{n} = \eta_0\,\frac{n}{\epsilon_{\text{rel}}} \end{aligned} \tag{1.3.10}$$

For a non-magnetic material, we have $\mu = \mu_0$, or, $\mu_{\text{rel}} = 1$, and the impedance becomes simply $\eta = \eta_0/n$, a relationship that we will use extensively in this book.

More generally, constitutive relations may be inhomogeneous, anisotropic, nonlinear, frequency dependent (dispersive), or all of the above. In *inhomogeneous materials*, the permittivity ϵ depends on the location within the material:

$$\boldsymbol{D}(\mathbf{r},t) = \epsilon(\mathbf{r})\boldsymbol{E}(\mathbf{r},t)$$

In *anisotropic materials*, ϵ depends on the x, y, z direction and the constitutive relations may be written component-wise in matrix (or tensor) form:

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \tag{1.3.11}$$

Anisotropy is an inherent property of the atomic/molecular structure of the dielectric. It may also be caused by the application of external fields. For example, conductors and plasmas in the presence of a constant magnetic field—such as the ionosphere in the presence of the Earth's magnetic field—become anisotropic (see for example, Problem 1.10 on the Hall effect.)

In *nonlinear materials*, ϵ may depend on the magnitude E of the applied electric field in the form:

$$\boldsymbol{D} = \epsilon(E)\boldsymbol{E}, \qquad \text{where} \qquad \epsilon(E) = \epsilon + \epsilon_2 E + \epsilon_3 E^2 + \cdots \tag{1.3.12}$$

Nonlinear effects are desirable in some applications, such as various types of electro-optic effects used in light phase modulators and phase retarders for altering polarization. In other applications, however, they are undesirable. For example, in optical fibers nonlinear effects become important if the transmitted power is increased beyond a few milliwatts. A typical consequence of nonlinearity is to cause the generation of higher harmonics, for example, if $E = E_0 e^{j\omega t}$, then Eq. (1.3.12) gives:

$$D = \epsilon(E)E = \epsilon E + \epsilon_2 E^2 + \epsilon_3 E^3 + \cdots = \epsilon E_0 e^{j\omega t} + \epsilon_2 E_0^2 e^{2j\omega t} + \epsilon_3 E_0^3 e^{3j\omega t} + \cdots$$

Thus the input frequency ω is replaced by $\omega, 2\omega, 3\omega$, and so on. In a multi-wavelength transmission system, such as a wavelength division multiplexed (WDM) optical fiber system carrying signals at closely-spaced carrier frequencies, such nonlinearities will cause the appearance of new frequencies which may be viewed as crosstalk among the original channels. For example, if the system carries frequencies ω_i, $i = 1, 2, \dots$, then the presence of a cubic nonlinearity E^3 will cause the appearance of the frequencies $\omega_i \pm \omega_j \pm \omega_k$. In particular, the frequencies $\omega_i + \omega_j - \omega_k$ are most likely to be confused as crosstalk because of the close spacing of the carrier frequencies.

Materials with a *frequency-dependent* dielectric constant $\epsilon(\omega)$ are referred to as *dispersive*. The frequency dependence comes about because when a time-varying electric field is applied, the polarization response of the material cannot be instantaneous. Such *dynamic* response can be described by the convolutional (and causal) constitutive relationship:

$$\boldsymbol{D}(\mathbf{r},t) = \int_{-\infty}^{t} \epsilon(t-t')\boldsymbol{E}(\mathbf{r},t')\,dt' \tag{1.3.13}$$

which becomes multiplicative in the frequency domain:

$$\boxed{\boldsymbol{D}(\mathbf{r},\omega) = \epsilon(\omega)\boldsymbol{E}(\mathbf{r},\omega)} \tag{1.3.14}$$

All materials are, in fact, dispersive. However, $\epsilon(\omega)$ typically exhibits strong dependence on ω only for certain frequencies. For example, water at optical frequencies has refractive index $n = \sqrt{\epsilon_{\text{rel}}} = 1.33$, but at RF down to dc, it has $n = 9$.

In Sec. 1.9, we discuss simple models of $\epsilon(\omega)$ for dielectrics, conductors, and plasmas, and clarify the nature of Ohm's law:

$$\boxed{\boldsymbol{J} = \sigma \boldsymbol{E}} \qquad \text{(Ohm's law)} \tag{1.3.15}$$

In Sec. 1.10, we discuss the Kramers-Kronig dispersion relations, which are a direct consequence of the causality of the time-domain dielectric response function $\epsilon(t)$.

One major consequence of material dispersion is *pulse spreading*, that is, the progressive widening of a pulse as it propagates through such a material. This effect limits the data rate at which pulses can be transmitted. There are other types of dispersion, such as *intermodal dispersion* in which several modes may propagate simultaneously, or *waveguide dispersion* introduced by the confining walls of a waveguide.

There exist materials that are both nonlinear and dispersive that support certain types of non-linear waves called *solitons*, in which the spreading effect of dispersion is exactly canceled by the nonlinearity. Therefore, soliton pulses maintain their shape as they propagate in such media [1151,850,851].

More complicated forms of constitutive relationships arise in chiral and gyrotropic media and are discussed in Chap. 4. The more general bi-isotropic and bi-anisotropic media are discussed in [30,73].

In Eqs. (1.1.1), the densities $\rho, \boldsymbol{J}$ represent the *external* or *free* charges and currents in a material medium. The induced polarization $\boldsymbol{P}$ and magnetization $\boldsymbol{M}$ may be made explicit in Maxwell's equations by using constitutive relations:

$$\boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P}, \quad \boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M}) \tag{1.3.16}$$

Inserting these in Eq. (1.1.1), for example, by writing $\nabla \times \boldsymbol{B} = \mu_0 \nabla \times (\boldsymbol{H} + \boldsymbol{M}) = \mu_0(\boldsymbol{J} + \dot{\boldsymbol{D}} + \nabla \times \boldsymbol{M}) = \mu_0(\epsilon_0 \dot{\boldsymbol{E}} + \boldsymbol{J} + \dot{\boldsymbol{P}} + \nabla \times \boldsymbol{M})$, we may express Maxwell's equations in terms of the fields $\boldsymbol{E}$ and $\boldsymbol{B}$:

$$\begin{aligned}
\nabla \times \boldsymbol{E} &= -\frac{\partial \boldsymbol{B}}{\partial t} \\
\nabla \times \boldsymbol{B} &= \mu_0 \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} + \mu_0 \big[\boldsymbol{J} + \frac{\partial \boldsymbol{P}}{\partial t} + \nabla \times \boldsymbol{M}\big] \\
\nabla \cdot \boldsymbol{E} &= \frac{1}{\epsilon_0}(\rho - \nabla \cdot \boldsymbol{P}) \\
\nabla \cdot \boldsymbol{B} &= 0
\end{aligned} \tag{1.3.17}$$

We identify the current and charge densities due to the polarization of the material as:

$$\boxed{\boldsymbol{J}_{\text{pol}} = \frac{\partial \boldsymbol{P}}{\partial t}, \quad \rho_{\text{pol}} = -\nabla \cdot \boldsymbol{P}} \qquad \text{(polarization densities)} \tag{1.3.18}$$

Similarly, the quantity $\boldsymbol{J}_{\text{mag}} = \nabla \times \boldsymbol{M}$ may be identified as the magnetization current density (note that $\rho_{\text{mag}} = 0$.) The total current and charge densities are:

$$\begin{aligned}
\boldsymbol{J}_{\text{tot}} &= \boldsymbol{J} + \boldsymbol{J}_{\text{pol}} + \boldsymbol{J}_{\text{mag}} = \boldsymbol{J} + \frac{\partial \boldsymbol{P}}{\partial t} + \nabla \times \boldsymbol{M} \\
\rho_{\text{tot}} &= \rho + \rho_{\text{pol}} = \rho - \nabla \cdot \boldsymbol{P}
\end{aligned} \tag{1.3.19}$$

and may be thought of as the *sources* of the fields in Eq. (1.3.17). In Sec. 14.6, we examine this interpretation further and show how it leads to the Ewald-Oseen extinction theorem and to a microscopic explanation of the origin of the refractive index.

Negative Index Media

Maxwell's equations do not preclude the possibility that one or both of the quantities ϵ, μ be negative. For example, plasmas below their plasma frequency, and metals up to optical frequencies, have $\epsilon < 0$ and $\mu > 0$, with interesting applications such as surface plasmons (see Sec. 8.5).

Isotropic media with $\mu < 0$ and $\epsilon > 0$ are more difficult to come by [130], although examples of such media have been fabricated [358].

Negative-index media, also known as left-handed media, have ϵ, μ that are simultaneously negative, $\epsilon < 0$ and $\mu < 0$. Veselago [353] was the first to study their unusual electromagnetic properties, such as having a negative index of refraction and the reversal of Snel's law.

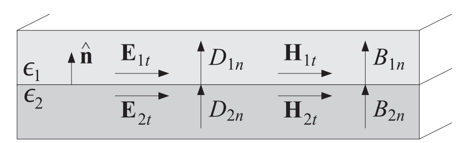
The novel properties of such media and their potential applications have generated a lot of research interest [353–434]. Examples of such media, termed "metamaterials", have been constructed using periodic arrays of wires and split-ring resonators, [359] and by transmission line elements [392–394,414,427], and have been shown to exhibit the properties predicted by Veselago.

When $\epsilon_{\text{rel}} < 0$ and $\mu_{\text{rel}} < 0$, the refractive index, $n^2 = \epsilon_{\text{rel}}\mu_{\text{rel}}$, must be defined by the negative square root $n = -\sqrt{\epsilon_{\text{rel}}\mu_{\text{rel}}}$. Because then $n < 0$ and $\mu_{\text{rel}} < 0$ will imply that the characteristic impedance of the medium $\eta = \eta_0 \mu_{\text{rel}}/n$ will be positive, which as we will see later implies that the energy flux of a wave is in the same direction as the direction of propagation. We discuss such media in Sections 2.12, 7.16, and 8.6.

1.4 Boundary Conditions

The boundary conditions for the electromagnetic fields across material boundaries are given below:

$$\boxed{\begin{aligned}
\boldsymbol{E}_{1t} - \boldsymbol{E}_{2t} &= 0 \\
\boldsymbol{H}_{1t} - \boldsymbol{H}_{2t} &= \boldsymbol{J}_s \times \hat{\mathbf{n}} \\
D_{1n} - D_{2n} &= \rho_s \\
B_{1n} - B_{2n} &= 0
\end{aligned}} \tag{1.4.1}$$



where $\hat{\mathbf{n}}$ is a unit vector normal to the boundary pointing from medium-2 into medium-1. The quantities $\rho_s, \mathbf{J}_s$ are any external *surface charge* and *surface current* densities on the boundary surface and are measured in units of [coulomb/m^2] and [ampere/m].

In words, the *tangential* components of the $\boldsymbol{E}$-field are continuous across the interface; the difference of the *tangential* components of the $\boldsymbol{H}$-field are equal to the surface current density; the difference of the *normal* components of the flux density $\boldsymbol{D}$ are equal to the surface charge density; and the *normal* components of the magnetic flux density $\boldsymbol{B}$ are continuous.

The D_n boundary condition may also be written a form that brings out the dependence on the polarization surface charges:

$$(\epsilon_0 E_{1n} + P_{1n}) - (\epsilon_0 E_{2n} + P_{2n}) = \rho_s \quad \Rightarrow \quad \epsilon_0 (E_{1n} - E_{2n}) = \rho_s - P_{1n} + P_{2n} = \rho_{s,\text{tot}}$$

The total surface charge density will be $\rho_{s,\text{tot}} = \rho_s + \rho_{1s,\text{pol}} + \rho_{2s,\text{pol}}$, where the surface charge density of polarization charges accumulating at the surface of a dielectric is seen to be ($\hat{\mathbf{n}}$ is the outward normal from the dielectric):

$$\boxed{\rho_{s,\text{pol}} = P_n = \hat{\mathbf{n}} \cdot \boldsymbol{P}} \tag{1.4.2}$$

The relative directions of the field vectors are shown in Fig. 1.4.1. Each vector may be decomposed as the sum of a part tangential to the surface and a part perpendicular to it, that is, $\boldsymbol{E} = \boldsymbol{E}_t + \boldsymbol{E}_n$. Using the vector identity,

$$\boldsymbol{E} = \hat{\mathbf{n}} \times (\boldsymbol{E} \times \hat{\mathbf{n}}) + \hat{\mathbf{n}}(\hat{\mathbf{n}} \cdot \boldsymbol{E}) = \boldsymbol{E}_t + \boldsymbol{E}_n \tag{1.4.3}$$

we identify these two parts as:

$$\boldsymbol{E}_t = \hat{\mathbf{n}} \times (\boldsymbol{E} \times \hat{\mathbf{n}}), \qquad \boldsymbol{E}_n = \hat{\mathbf{n}}(\hat{\mathbf{n}} \cdot \boldsymbol{E}) = \hat{\mathbf{n}} E_n$$

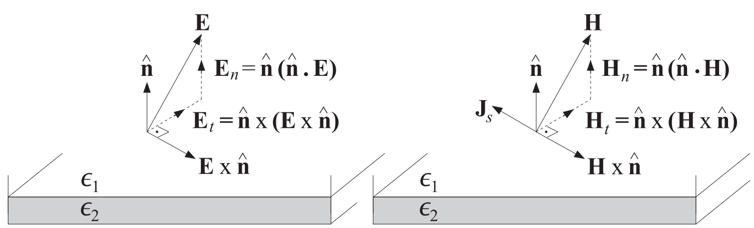


Fig. 1.4.1 Field directions at boundary.

Using these results, we can write the first two boundary conditions in the following vectorial forms, where the second form is obtained by taking the cross product of the first with $\hat{\mathbf{n}}$ and noting that $\boldsymbol{J}_s$ is purely tangential:

$$\boxed{\begin{aligned} \hat{\mathbf{n}} \times (\boldsymbol{E}_1 \times \hat{\mathbf{n}}) - \hat{\mathbf{n}} \times (\boldsymbol{E}_2 \times \hat{\mathbf{n}}) &= 0 \\ \hat{\mathbf{n}} \times (\boldsymbol{H}_1 \times \hat{\mathbf{n}}) - \hat{\mathbf{n}} \times (\boldsymbol{H}_2 \times \hat{\mathbf{n}}) &= \boldsymbol{J}_s \times \hat{\mathbf{n}} \end{aligned}} \quad \text{or,} \quad \boxed{\begin{aligned} \hat{\mathbf{n}} \times (\boldsymbol{E}_1 - \boldsymbol{E}_2) &= 0 \\ \hat{\mathbf{n}} \times (\boldsymbol{H}_1 - \boldsymbol{H}_2) &= \boldsymbol{J}_s \end{aligned}} \tag{1.4.4}$$

The boundary conditions (1.4.1) can be derived from the integrated form of Maxwell's equations if we make some additional regularity assumptions about the fields at the interfaces.

In many interface problems, there are no externally applied surface charges or currents on the boundary. In such cases, the boundary conditions may be stated as:

$$\boxed{\begin{aligned} \boldsymbol{E}_{1t} &= \boldsymbol{E}_{2t} \\ \boldsymbol{H}_{1t} &= \boldsymbol{H}_{2t} \\ D_{1n} &= D_{2n} \\ B_{1n} &= B_{2n} \end{aligned}} \qquad \text{(source-free boundary conditions)} \tag{1.4.5}$$

1.5 Currents, Fluxes, and Conservation Laws

The electric current density $\boldsymbol{J}$ is an example of a *flux vector* representing the flow of the electric charge. The concept of flux is more general and applies to any quantity that flows.† It could, for example, apply to energy flux, momentum flux (which translates into pressure force), mass flux, and so on.

In general, the flux of a quantity Q is defined as the amount of the quantity that flows (perpendicularly) through a unit surface in unit time. Thus, if the amount ΔQ flows through the surface ΔS in time Δt, then:

$$J = \frac{\Delta Q}{\Delta S \Delta t} \qquad \text{(definition of flux)} \tag{1.5.1}$$

When the flowing quantity Q is the electric charge, the amount of current through the surface ΔS will be $\Delta I = \Delta Q/\Delta t$, and therefore, we can write $J = \Delta I/\Delta S$, with units of [ampere/m^2].

The flux is a vectorial quantity whose direction points in the direction of flow. There is a fundamental relationship that relates the flux vector $\boldsymbol{J}$ to the transport velocity $\boldsymbol{v}$ and the volume density ρ of the flowing quantity:

$$\boxed{\boldsymbol{J} = \rho \boldsymbol{v}} \tag{1.5.2}$$

This can be derived with the help of Fig. 1.5.1. Consider a surface ΔS oriented perpendicularly to the flow velocity. In time Δt, the entire amount of the quantity contained in the cylindrical volume of height $v\Delta t$ will manage to flow through ΔS. This amount is equal to the density of the material times the cylindrical volume $\Delta V = \Delta S(v\Delta t)$, that is, $\Delta Q = \rho \Delta V = \rho\, \Delta S\, v\Delta t$. Thus, by definition:

$$J = \frac{\Delta Q}{\Delta S \Delta t} = \frac{\rho\, \Delta S\, v \Delta t}{\Delta S \Delta t} = \rho v$$

When $\boldsymbol{J}$ represents electric current density, we will see in Sec. 1.9 that Eq. (1.5.2) implies Ohm's law $\boldsymbol{J} = \sigma \boldsymbol{E}$. When the vector $\boldsymbol{J}$ represents the energy flux of a propagating electromagnetic wave and ρ the corresponding energy per unit volume, then because the speed of propagation is the velocity of light, we expect that Eq. (1.5.2) will take the form:

$$J_{\text{en}} = c\rho_{\text{en}} \tag{1.5.3}$$

†In this sense, the terms electric and magnetic "flux densities" for the quantities $\boldsymbol{D}, \boldsymbol{B}$ are somewhat of a misnomer because they do not represent anything that flows.