

COMPUTATIONAL  
INVERSE TECHNIQUES  
*in*  
NONDESTRUCTIVE  
EVALUATION

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G.R. Liu  
X. Han



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# *Dedication*

*To Zuona  
Yun, Kun, Run,  
and my family  
for the time they gave to me*

**G. R. Liu**

*To Zhenglin, Weiqi  
and my family for their support*

*To my mentor, Dr. Liu  
for his guidance*

**X. Han**

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## *Preface*

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In the past 2 decades, inverse problems have been one of the most important focal areas of research in engineering. Advances in computational numerical methods and techniques in computer hardware and software in so-called soft computing have enabled inverse techniques to become a powerful tool for practical engineering problems. However, for many researchers and engineers, inverse analysis is still a distant topic because of a frightening and mysterious image of the difficulty of grasping and dealing with its concepts.

In the early 1980s, G. R. Liu encountered the first inverse problem in his years of postgraduate study for the characterization of composite laminates. Dr. Liu was alarmed and confused by the flood of unfamiliar terminologies related to inverse problems, such as ill-posedness, regularization, stability, uniqueness, etc. In fact, he worried so much about the possibility of doing things improperly that he finally gave up pursuing the problem in the context of inverse analysis. He solved the problem by cutting the composite laminates into pieces and measuring the mechanical and thermal properties using traditional tensile machines and thermal measurement equipment — a destructive, time consuming, problematic, but conventionally accepted approach. Back then, he wished for a book like this one to guide his research work so that he could conduct it in a more advanced manner.

The next time Dr. Liu got the courage to face the same inverse problem was in 1997, when he had a set of good forward solvers for waves in composite laminates. This time he decided to put these terminologies aside and go straight ahead to formulate and use the optimization tools to solve the problem. He managed to obtain the solution without too much difficulty, but with many mistakes. Based on confidence built upon the first trial study, he then turned to look at these terminologies, and found that they were, in fact, walls that scared people away.

The best way to break open these walls is to solve an inverse problem first following general knowledge, and then dealing with the issues as they occur. Slowly, experience will accumulate and tricks and techniques will be learned so that increasing numbers of inverse problems can be solved. This means learning or training was difficult before, but much easier and particularly useful now because these practices can be performed in a PC environment, to which almost everyone has access.

The authors have learned some inverse techniques through the aforementioned hard way; they decided to put their experiences in this book on how inverse problems of mechanics can be formulated and solved and the possible issues important for successful inverse analysis. They are committed to putting all these materials in a very simple and easy-to-understand form, as well

as using the simplest examples to reveal true meanings of these abstruse terminologies and the mechanisms of some important phenomena. Many example problems and practical engineering problems are presented, together with many numerical tests as well as some experimental verification.

The authors hope that this book can help readers face inverse problems comfortably and tackle them with ease, without being frightened off. The truth is that many engineering inverse problems are not that difficult because they can be well posed if they are properly formulated with a sound experimental strategy.

Properly formulating and solving an inverse problem demands that the analyst have (1) very good understanding of the physical problem, (2) good experimental strategy and quality measurement data, and (3) most importantly, effective computational techniques. Without a good understanding of the physics of the problem, basically nothing can be done. This book will not help much in this context, except to emphasize the importance of this understanding. Quality measurement data are essential because they will decide the quality of the solution of the inverse problem. This includes not only the accuracy of the experimental (or test or observational) data, but also the precise knowledge of the characteristics of the measurement data in terms of the noise content (noise level, frequency, etc.). Apart from modern high-tech experimental equipment, acquiring such quality experimental data depends highly on understanding the physics involved in the problem and the process of measurement. Although, this book will cover some of the issues in measuring wave and vibration responses of structures, they are not its focus.

This book emphasizes the key to solution of any practical and complex inverse problems: computational techniques. These techniques concern how to obtain what is needed from given experimental data efficiently and accurately. Without the computer and effective computational techniques, it is not possible to perform a decent inverse analysis of a complex engineering problem. Forward solver is also very important, but this book generally assumes that a reliable forward solver to the physical problem is available. Thus, only sources of and a brief introduction to forward solvers are provided here. It is the task of the analyst to use these forward solvers properly and produce reliable results for the inverse analysis — by no means an easy task, but not the focus of this volume. Readers may refer to earlier books by Dr. Liu or other related literature.

The authors' work in the area of inverse analysis has been profoundly influenced and guided by many existing works reported in the open literature, which are partially listed in the references. Without those significant contributions to this area, this book would not exist. The authors would like to thank all the authors of the excellent papers and books published in areas related to this book's topic.

Many colleagues and students have supported and contributed to the writing of this book. Dr. Liu expresses sincere thanks to all of them, with special appreciation to Y.G. Xu, Z.L. Yang, Z.P. Wu, S.I. Ishak, H.M. Shang,

S.P. Lim, W.B. Ma, Irwan Bin Karim, S.C. Chen, and H.J. Ma. Many of them have contributed examples to this book in addition to their hard work in carrying out a number of projects related to inverse problems.

Finally, the authors would also like to thank A\*STAR, Singapore, for its partial financial sponsorship for research projects related to the topic of this book that were undertaken by the authors and their teams.

**G.R. Liu and X. Han**

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## Authors

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**Dr. G.R. Liu** received his Ph.D. from Tohoku University, Japan, in 1991. He was a postdoctoral fellow at Northwestern University, Evanston, Illinois. He is currently the director of the center for advanced computations in engineering science (ACES), National University of Singapore and an associate professor in the Department of Mechanical Engineering, National University of Singapore. He is also currently the president of the Association for Computational Mechanics (Singapore). Dr. Liu has provided consultation services to many national and international organizations and authored more than 300 technical publications, including more than 180 international journal papers. He has written five books, including the popular book *Mesh-Free Method: Moving beyond the Finite Element Method*. He serves as an editor and a member of editorial boards of five scientific journals. Dr. Liu is the recipient of the Outstanding University Researchers Award, the Defense Technology Prize, and the Silver Award at CrayQuest (nationwide competition). His research interests include computational mechanics, mesh-free methods, nanoscale computation, microbio-system computation, vibration and wave propagation in composites, mechanics of composites and smart materials, inverse problems, and numerical analysis.



**Dr. X. Han** obtained his bachelor's and master's degrees in engineering mechanics from Harbin Institute of Technology, China, in 1990 and 1997, respectively, and his doctorate in mechanical engineering from National University of Singapore in 2001. He was a research fellow at the School of Mechanical and Production Engineering, Nanyang Technology University, Singapore. Dr. Han has been working on the development of numerical analysis techniques for wave propagation problems and computational inverse techniques. He is currently the manager of the center for advanced computations in engineering science (ACES), Department of Mechanical Engineering, National University of Singapore. Dr. Han's research interests include structural dynamics of advanced composite and smart materials, inverse problems and numerical analysis. He is the author or co-author of approximately 30 referenced journal papers.





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## *Introduction*

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### **1.1 Forward and Inverse Problems Encountered in Structural Systems**

In engineering, computer-aided design (CAD) tools are used to design advanced structural systems. Computational simulation techniques are often used in such tools to calculate the displacement, deflection, strains, stresses, natural frequencies, and vibration modes, etc. in the structural system for given loading, initial and boundary conditions, geometrical configuration, material properties, etc. of the structure. These types of problems are called *forward problems* and are often governed by *ordinary* or *partial differential equations* (ODE or PDE) with unknown *field variables*. For structure mechanics problems, the field variable is basically the displacements; the constants in the ODE or PDE and problem domain are known *a priori*. The source or the cause of the problem or phenomenon governed by the ODE or PDE and the relevant initial and boundary conditions are also known. To solve a forward problem is, in fact, to solve the ODE or PDE subjected to these initial and boundary conditions.

Many solution procedures, especially the computational procedures, have been developed, such as:

- Finite difference method (FDM; see e.g., Hirsch, 1988; Anderson, 1995)
- Finite element method (FEM; see e.g., Zienkiewicz and Taylor, 2000; Liu and Quek, 2003)
- Strip element method (Section 10.3)
- Boundary element method (BEM; see e.g., Brebbia et al., 1984)
- FEM/BEM (see e.g., Liu, Achenbach et al., 1992)
- Mesh-free methods (see e.g., Liu, 2002a; Liu and Liu, 2003)
- Wave propagation solvers (see e.g., Liu and Xi, 2001)

These methods for solving forward problems have been well established, although the mesh-free methods are still in a stage of rapid development. Using these methods, the displacements in the structure and then the strains