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WITTGENSTEIN, FINITISM, AND THE FOUNDATIONS OF MATHEMATICS

Mathieu Marion

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PREFACE

During his life, Wittgenstein published only one book, the *Tractatus Logico-Philosophicus* (1921), one paper, 'Some Remarks on Logical Form' (1929), one page-long book review (1913), and one letter to the editor (1930). But he left behind an extensive *Nachlass*. From the amount of secondary literature devoted to it, it is surprising to discover that Wittgenstein wrote more on philosophy of mathematics than on any other subject. That Wittgenstein's remarks on mathematics did not receive their fair share of attention is not to be explained only by the fact that it is a comparatively unpopular topic. When Wittgenstein's *Remarks on the Foundations of Mathematics* (*RFM*) were first published in 1956, reviewers' assessments were negative. For example, the logician Georg Kreisel, a close friend who had frequent discussions with Wittgenstein in the early 1940s, ended his review of the book with these much-quoted words: 'it seems to me to be a surprisingly insignificant product of a sparkling mind' (Kreisel 1958a: 158). Since specialists were in agreement in their negative assessment, followers and commentators of Wittgenstein simply hived off issues in philosophy of mathematics from those concerning language and psychology, more or less assuming that, although Wittgenstein may have erred when tackling issues in mathematical logic and foundations of mathematics, this was of no consequence to the rest of his philosophy: they could then continue with the business of interpreting these other parts in isolation. To my mind, this is ultimately as unacceptable as it would be for someone to interpret Frege's or Russell's philosophy in ignorance of their work in mathematical logic and the foundations of mathematics.

A variety of factors explain the negative reception of Wittgenstein's remarks on the philosophy of mathematics in the 1950s. First, the book comprised remarks selected by the editors (G. E. M. Anscombe, R. Rhees, and G. H. von Wright) from various manuscripts dating from 1937 to 1944 and it is clear that editorial choices, which produced a truncated version of the original text, have hindered rather than helped us to understand Wittgenstein's thoughts. He was an undoubtedly difficult writer, but when whole paragraphs are cut out in between two published remarks without any indications to that

effect, the reader may be understandably puzzled by seemingly bizarre jumps in Wittgenstein's thought. Secondly, access to Wittgenstein's writings was very limited at the time: of the posthumous writings, only the *Philosophical Investigations* (*PI*) and *The Blue and Brown Books* (*BB*) had been published. Wittgenstein's remarks, when they did not relate directly to *Philosophical Investigations*, published a few years earlier, often seemed to be coming out of the blue. One had to wait almost a decade for the first major work of the transitional period (from Wittgenstein's return to Cambridge in 1929 to the first draft of the *Blue Book* in 1933), where the later philosophy of mathematics has its source, to be published. This was *Philosophical Remarks* (*PR*) in 1964, the English translation appearing in 1975. Thirdly, Wittgenstein scholarship was simply in its infancy: it was difficult to get a good grasp of Wittgenstein's later philosophy, let alone make sense of what he had to say on mathematics. Finally, the intellectual climate of the late 1950s did not make for an easy reception of Wittgenstein's ideas: results such as Gödel's on incompleteness, Tarski's on truth, and Gödel's and Cohen's on the independence of the Axiom of Choice and the Continuum Hypothesis from Zermelo-Fraenkel's set theory had given much credence to radically non-constructive forms of Platonism in philosophy of mathematics, against which Wittgenstein's remarks seem prima facie directed. To make a long story short, if the picture of Wittgenstein presented in this book is on the whole on the right lines, the ideology of the 1950s was simply very much against Wittgenstein. For all these reasons, it is not surprising that those who read him easily misrepresented his views and rejected them flatly.

This early negative assessment of Wittgenstein's philosophy of mathematics and the concomitant idea that this part of his philosophical output is at any rate of little interest have become firmly implanted. But the situation is changing. To begin with, many more of Wittgenstein's posthumous writings are now easily available in print, and there are now shelves full of commentaries. We can safely say that our understanding of his thought has improved markedly, thanks to the work of commentators such as Gordon Baker and Peter Hacker, Anthony Kenny, Brian McGuinness, and David Pears. Moreover, the focus in mathematical logic has recently shifted considerably towards constructivism, with the growing interest in connections with theoretical computer science (especially with complexity theory): logicians are now interested not only in constructive

proofs but in an even more restricted class of proofs, those that are said to be 'feasible' because they provide an algorithm which is polynomial-time computable, so that the proof can be run on a computer. In this context, issues relating to finitism and the strict finitist philosophy of mathematics, to which the name of Wittgenstein was linked by the reviewers mentioned above, have resurfaced. It is thus time slowly to piece together the various strands of Wittgenstein's philosophy of mathematics and aim for a reassessment.

One thing, however, needs to change. As pointed out, only some of Wittgenstein's later works had been published in the 1950s: the *Philosophical Investigations* in 1953, the *Remarks on the Foundations of Mathematics* in 1956, and the *Blue and Brown Books* in 1958. In postwar Oxford in particular, this editorial policy led to the development of interpretations where the later Wittgenstein, with his strategic affinities with ordinary language philosophy, was favourably contrasted with the earlier Wittgenstein, who was seen to be going in the (terribly wrong) direction of logical positivism. Thus a distorting emphasis was put on Wittgenstein II versus Wittgenstein I; and between the two great philosophies there seems to have been that grey zone called the 'transitional' period, which was considered not to contain any idea worth a closer look. Such prejudices are still very strong today, especially among those whose ideas about Wittgenstein were formed in the 1950s. True, it is better exegetically speaking to look at Wittgenstein's arguments as being directed against views held by his former self. He himself tells us in the preface to the *Philosophical Investigations* to read that work in conjunction with his *Tractatus Logico-Philosophicus* in order that we see the mistakes in the latter. But in no way does this justify ignorance of the transitional writings. To take a personal example, I began studying Wittgenstein by struggling with the *Remarks on the Foundations of Mathematics* (this time with a new enlarged edition, which still, inexplicably, does not contain the full version of the original texts) and with the topic of the day in the 1980s, the rule-following argument. I had in front of me an immense literature where almost no one cared about the development of Wittgenstein's thoughts. Hardly anyone tried to find out what it was exactly in his earlier views that he was reacting against or how his arguments took shape: it is as if Wittgenstein had a conversion. What was needed instead was a reading of his transitional writings which would provide us with an